N-body Simulation Tutorial

**Introduction**

Today, we are using a small direct N-body integrator written in *Python* to simulate N-body interactions like three-body interactions. We will observe their motions and learn about the dynamical interactions between stars in globular clusters, galaxies and in the Universe. We will also learn that two-body problem is easy to solve; Newton solved it about 300 years ago. However, three-body problem is complicated, and it requires computers to solve it approximately.

**About the N-body Simulator**

The N-body simulator uses the ***leapfrog*** method to calculate the forces between the stars, their velocities and positions as a function of time. In theory, if we know the masses, initial positions and velocities of all the stars, we can calculate their velocities and positions at the following times, using Newton’s second law .

For example, to approximately calculate and , if the mass of a star is , and the total net force on this star is , the acceleration of the star is

.

Taking small time intervals (e.g. 1 second), the new velocity at a new time can be written as

,

where is the initial velocity of the star. And the new position at time is

The leapfrog method calculates the velocities at half the time step , and the positions at one time step . In doing this, the simulation is more stable and it is easier to conserve the total energy of the stars. Total energy and angular momentum of the system with N stars should be conserved.

All the codes are written in ***Python***.

**Running the N-body Simulator**

*Step 1*: Go to <https://mybinder.org/v2/gh/claireshiye/GK12/master>, and then click open the folder **N-body Simulator**.

*Step 2*: First open the **Nbody\_plot.ipynb** file, on the menu at the top, click **cell**, and then click **Run all**.A box asking for input will show, type in **example\_binary**. This is a folder that contains a data file made for the dynamical interactions of two point-like stars with the same masses. Watch the animation.

What can you see? You have a binary star system! The two stars orbit each other while moving together at a certain direction. They move at that direction because their total initial momentum points toward that direction. But their trajectories and orbits have a pattern: they stay in the same binary orbit.

*Step 3*: Now click **cell**, and then click **Run all** again, but this time type in **example\_triple**.

What can you see this time? The three stars do not have stable orbits (by stable I mean the orbits have the same pattern and do not look like a mess ). This is very interesting. Simply by adding one star to the interaction, we have a much more complicated system than the binary system we saw in step 2.

*Step 4*: Now you are going to simulate a three-body interaction. Go to **Home,** open file **N-body\_simulator\_leapfrog.ipynb**, click **cell**, then click **Run all**. A few boxes will show asking for input, type in the numbers as follow,

Number of particles: 3

Number of steps (divisible by 100): 10000

Number of dimensions (2 or 3): 3

Specify the timestep: 0.01

Randomly generate masses?: yes

Randomly generate particle positions?: yes

(Note: Your input of the number of particles decides how many stars are going to be in the simulation. Number of steps tells the code how many time steps to take. Number of dimensions sets the dimension of the simulation, 2 is 2-dimension (x & y axis), and 3 is 3-dimension (x, y & z axis). Time step is from above in *About the N-body Simulator*. The smaller the time step , the more accurate the simulation is. For this part, let’s generate the masses and initial star positions randomly (the code will pick the masses and initial star positions for you within a range). If you put in ‘no’ in both “Randomly generate masses?” and “Randomly generate particle positions”, you will need to manually put in the masses and initial positions of all the stars. We will do this later.)

After you put in all the numbers, the code will run, and it will finish in 1-2 mins. It will generate a new folder with year-date-time as the folder name.

*Step 5*: Now go to the **Home** folder and find the folder you just made. Go back to **Nbody\_plot.ipynb**. Repeat step 1, but this time type in the new folder name instead of example\_binary or example\_triple. Watch the animation.

*Step 6*: Play with the simulator. Repeat step 4, but this time choose your own input.

Choose the ‘number of particles’ between 2-5,

‘number of steps’ between 5000 to 10000,

‘number of dimensions’ 2 or 3,

‘timestep’ 0.005, 0.01 or 0.02,

‘randomly generate masses?’ yes or no, if no, type in the masses of the stars in the range 1-10,

‘randomly generate positions’ yes (suggested).

Repeat step 5, this time type in the name of the folder you just made (not the one in step 5). Do you get a system that looks interesting? If the stars are doing something weird (for example, the trajectories are not smooth curves but step-like), try put in different numbers.